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Temperature crossovers in cuprates

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Abstract. We study the temperature crossovers seen in the magnetic and transport properties of cuprates using a nearly antiferromagnetic Fermi-liquid model (NAFLM). We distinguish between underdoped and overdoped systems on the basis of their low-frequency magnetic behaviour and so classify the optimally doped cuprates as a special case of the underdoped cuprates. For the overdoped cuprates, we find, in agreement with earlier work, mean-field $z = 2$ behaviour of the magnetic variables associated with the fact that the damping rate of their spin fluctuations is essentially independent of temperature, while the resistivity exhibits a crossover from Fermi-liquid behaviour at low temperature to linear-in-T behaviour above a certain temperature T_0. We demonstrate that above T_0 the proximity of the quasiparticle Fermi surface to the magnetic Brillouin zone boundary brings about the measured linear-in-T resistivity. For the underdoped cuprates we argue that the sequence of crossovers identified by Barzykin and Pines in the low-frequency magnetic behaviour (from mean-field $z = 2$ behaviour at high temperatures, $T > T_{cr}$, to non-universal $z = 1$ scaling behaviour at intermediate temperatures, $T_* < T < T_{cr}$, to pseudogap behaviour below T_*) reflects the development in the electronic structure of a precursor to a spin-density-wave state. This development begins at T_{cr} with a thermal evolution of the quasiparticle spectral weight which brings about temperature-dependent spin damping and ends at T_* where the Fermi surface has lost pieces near corners of the magnetic Brillouin zone. For $T_* < T < T_{cr}$ the resistivity is linear in T because this change in spectral weight does not affect the resistivity significantly; below T_* vertex corrections act to bring about the measured downturn in $(\rho(T) - \rho(0))/T$ and approximately quadratic-in-T resistivity for $T \ll T_*$.

1. Introduction

Over the past few years, it has become increasingly clear to the 'high-T_c' community that the mechanism of superconductivity in cuprates is directly related to their unusual properties in the normal state, particularly in the underdoped regime. The ^{63}Cu spin–lattice relaxation rate and spin-echo decay rate (Slichter 1994, Barzykin and Pines 1995, Curro *et al* 1996), uniform susceptibility (Johnson 1989), and in-plane and c-axis resistivity, ρ_{xx} and ρ_c (Ong 1990, Iye 1992) all demonstrate temperature dependences which over a wide temperature range are different from the predictions of Landau Fermi-liquid theory. The remarkable sequence of crossovers (from non-universal mean-field behaviour with a dynamical exponent $z = 2$ to $z = 1$ pseudoscaling behaviour to pseudogap behaviour) seen in magnetic experiments in the normal state of the optimally doped and underdoped cuprates are shown in figure 1. Characterizing and explaining this behaviour, which has counterparts in angle-resolved photoemission experiments (Marshall *et al* 1996, LaRosa *et al* 1996, Ding *et al* 1996), transport measurements (Hwang *et al* 1994), and optical experiments (Puchkov

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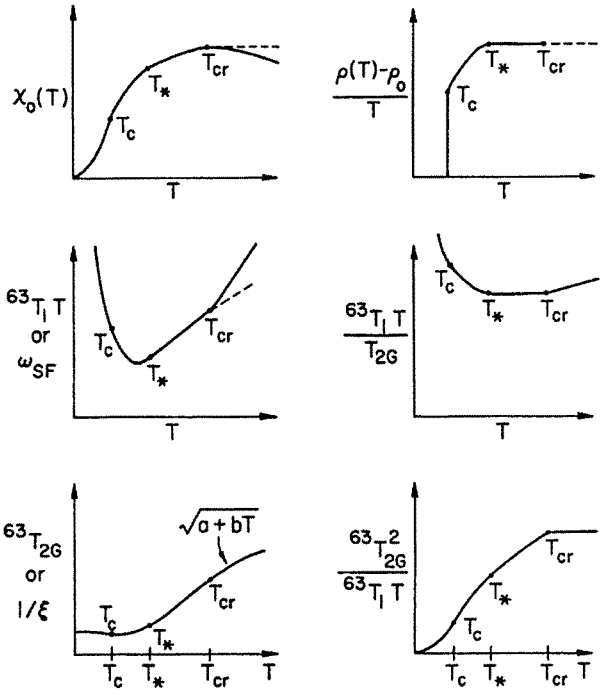

Figure 1. The schematic behaviour of various observables in underdoped cuprates (after Barzykin and Pines 1995). The temperature scales T_{cr} and T_* indicate crossovers which are defined and discussed in the text.

et al 1996) on the underdoped cuprates, is perhaps the major challenge currently facing the high-T_c community.

The effectively non-Landau-liquid behaviour at intermediate energy scales has stimulated intensive discussions on a possible violation of Fermi-liquid theory in cuprates (Anderson 1987, 1994). There is currently no consensus on whether Fermi-liquid behaviour is actually broken at $T = 0$. Some researchers believe that the ground state of underdoped and optimally doped cuprates is not a Fermi liquid: in particular, a non-Fermi-liquid ground state is a point of departure for gauge theories based on spin–charge separation (see, e.g., Lee and Nagaosa (1992), Nagaosa and Lee (1992), Altshuler *et al* (1995)), and for theories in which pairing is due to pair hopping between adjacent layers (Chakravarty and Anderson 1994). Another conjecture is that there exists an isolated zero-temperature quantum-critical point at a doping concentration near optimal doping (Varma 1996). On both sides of this point, it is proposed that the system behaves as a Fermi liquid at the lowest T, but exhibits a non-Fermi-liquid quantum-critical behaviour above the crossover temperature, which near optimal doping is assumed to be smaller than T_c.

The authors of the present paper have argued in favour of a third possibility, a nearly antiferromagnetic Fermi-liquid model (NAFLM) for cuprates (for a review see, e.g, Pines 1995). In this approach there is no spin–charge separation. One assumes that Fermi-liquid behaviour is not destroyed by fluctuations at any doping concentration, while the measured anomalous (non-Landau-like) spin and charge behaviour seen in the normal-state properties of the cuprates arises from a magnetic interaction between planar quasiparticles

which reflects the close approach of even an optimally doped system to antiferromagnetism. Initial support for the NAFLM came from the NMR experiments which clearly demonstrate the difference between Cu and O spin–lattice relaxation rates and establish the presence of strong temperature-dependent antiferromagnetic correlations (for a review see Slichter 1994). More recently the model has been shown to be also consistent with neutron scattering data which show that even at optimal doping, the spin fluctuations are peaked at a wavevector $\boldsymbol{Q}$ which is at or near (π, π) with a magnitude, $\chi_{\boldsymbol{Q}} \gg \chi_0$, and a half-width, $1/\xi$, which at low temperatures is considerably smaller than the inverse lattice spacing (Mason *et al* 1994, Bourges *et al* 1996, Zha *et al* 1996). Moreover, recent experiments have shown that even at optimal doping there still exist propagating spin waves at energies comparable to the exchange integral (Hayden *et al* 1996).

In this paper, we consider the extent to which the NAFLM can explain the temperature crossovers in magnetic and resistivity measurements in both overdoped and underdoped cuprates. We will show that the NAFLM yields a sequence of crossovers and changes in the uniform susceptibility, NMR relaxation rates and resistivity which are consistent with experiments. In particular, we will show how for the underdoped cuprates the Fermi-surface evolution which accompanies the development of a precursor to a spin-density-wave state gives rise to $z = 1$ scaling at intermediate temperatures, and the pseudogap behaviour at the lowest temperatures.

The paper is organized as follows. In the next section we review the NAFLM description of cuprates and its relationship with the underlying microscopic models with fermion–fermion interaction. In sections 3, 4 and 5 we discuss the physics of overdoped, underdoped and optimally doped cuprates, respectively. Finally, in section 6 we summarize our conclusions.

2. The nearly antiferromagnetic Fermi-liquid model

The canonical NAFLM model is in some respects a two-fluid model for cuprates: it is equivalent to assuming that there independently exist fermionic carriers *and* localized spins whose susceptibility is determined from fits to NMR experiments and is an input parameter in the theory. Fermions and spins are coupled by

$$\mathcal{H}_{int} = \sum_{\boldsymbol{q},\boldsymbol{k},\alpha,\beta} g_{\boldsymbol{q}}\, c^{\dagger}_{\boldsymbol{k}+\boldsymbol{q},\alpha}\, \boldsymbol{\sigma}_{\alpha,\beta}\, c_{\boldsymbol{k},\beta} \cdot \boldsymbol{S}_{\boldsymbol{q}} \tag{1}$$

where $g_{\boldsymbol{q}}$ is the momentum-dependent coupling constant, and $\boldsymbol{\sigma}_i$ are the Pauli matrices. The momentum dependence of the coupling constant is not relevant for our considerations, and for simplicity we will neglect it throughout the paper.

To second order in the coupling, the spin–fermion interaction gives rise to an effective pairing interaction between planar quasiparticles

$$V_{eff}(\boldsymbol{q}, \omega) = g^2 \chi(\boldsymbol{q}, \omega) \tag{2}$$

where $\chi(\boldsymbol{q}, \omega)$ is the susceptibility of localized spins. NMR and neutron scattering experiments clearly indicate that near optimal doping the spin fluctuations are overdamped at low energies with a dynamical structure factor peaked at a wavevector $\boldsymbol{Q}$ which is close to (π, π) and symmetry-related points. From general considerations, one can then write near $\boldsymbol{q} = \boldsymbol{Q}$ and $\omega = 0$ (Millis *et al* 1990, Barzykin *et al* 1993, Monthoux and Pines 1994b)

$$\chi(\boldsymbol{q}, \omega) = \frac{\chi_{\boldsymbol{Q}}}{1 + (\boldsymbol{q} - \boldsymbol{Q})^2 \xi^2 - \mathrm{i}\omega/\omega_{sf} - \omega^2 \xi^2 / c_{sw}^2} \tag{3}$$

where $\chi_Q = \alpha\xi^2$, ξ is the correlation length, c_{sw} is the spin-wave velocity, and $\omega_{sf} = c_{sw}^2/2\xi^2\gamma$ where γ is a damping rate. The fits to the NMR data show that the parameters ξ, χ_Q and ω_{sf} depend on temperature, while the scale factor α and the spin-wave velocity c_{sw} are virtually independent of T in the temperature range of NMR experiments (Barzykin and Pines 1995).

It is also essential that the fermions are *not* assumed to be free particles at $g = 0$, and their Green's function is well defined only near the Fermi surface where $G(k, \omega) = Z_k/(\omega - \epsilon_k)$. The dispersion, ϵ_k, is another input parameter in the theory. It is generally assumed to have the same form as in the tight-binding model with hopping between nearest and next-nearest neighbours on a square lattice:

$$\epsilon_k = -2t(\cos k_x + \cos k_y) - 4t' \cos k_x \cos k_y. \tag{4}$$

This form of ϵ_k (with $t' < 0$) is chosen to be consistent with the shape of the Fermi surface measured in photoemission experiments at around optimal doping (Si *et al* 1993). Explicit calculations show (Monthoux and Pines 1993) that at these doping levels, the interaction with spin fluctuations does not modify substantially the shape of the Fermi surface as long as the coupling constant remains roughly smaller than the quasiparticle damping rate.

In principle, the correlation length in (3) should be large enough compared to the interatomic spacing to ensure that the susceptibility is peaked at $\boldsymbol{Q}$. In practice (Pines 1995), it turns out that a correlation length of the order of a lattice spacing already yields an appreciable peak in $\chi(\boldsymbol{q}, \omega)$. Furthermore, the analytical expansion in $(\boldsymbol{q} - \boldsymbol{Q})^2$ and ω is valid only outside the fluctuation region for a magnetic phase transition. This region is however rather narrow and most experiments are performed outside it. A more essential point is that $\chi(\boldsymbol{q}, \omega)$ in equation (3) does not satisfy the sum rule for the local susceptibility, as the 3D integral $\int \mathrm{d}^2q\ \mathrm{d}\omega\ \chi(\boldsymbol{q}, \omega)$ diverges at the upper limit. One way to obtain convergence is to include higher-order terms in the expansion over momentum. Another way, which we will adopt here, is to introduce a sharp cut-off, C, in the integration over $c_{sw}(\boldsymbol{q} - \boldsymbol{Q})$. Due to the thermal dependence of ξ and ω_{sf} in equation (3), the cut-off parameter C also depends on temperature. On general grounds, the larger the correlation length, the smaller the cut-off scale. However, C does not vanish when the correlation length diverges.

Although the phenomenological NAFLM considers localized spins as an independent degree of freedom, equations (1) and (2) can be derived from microscopic considerations departing from, e.g., the one-band Hubbard model which contains only fermionic degrees of freedom (Shraiman and Siggia 1988, Schrieffer *et al* 1989, Chubukov and Frenkel 1992, Bulut *et al* 1993, Scalapino 1994, Kampf 1994, Sachdev *et al* 1995). In this approach, spin fluctuations appear as a collective mode of fermions. To obtain the coupling between two fermions and one spin fluctuation, as in equation (1), one has to dress the original four-fermion Hubbard interaction term by summing the RPA series in the particle–hole channel. As a result of the summation, the product of the two fermionic Green's functions is replaced by a spin susceptibility whose poles correspond to spin-fluctuation modes. The susceptibility thus obtained takes the general form

$$\chi(\boldsymbol{q}, \omega) = \frac{\tilde{\chi}(\boldsymbol{q}, \omega)}{1 - g\tilde{\chi}(\boldsymbol{q}, \omega)} \tag{5}$$

in which $\tilde{\chi}(\boldsymbol{q}, \omega)$ is the irreducible particle–hole susceptibility, and g (the same as in equation (1)) is a coupling constant for the pair interaction between fermions. For example, for on-site Hubbard interaction, $g = U$. The momentum and frequency dependence of the spin susceptibility obtained in the Hubbard-based approach is generally consistent with

the phenomenological predictions except that at any finite doping level the peak in the susceptibility is located at an incommensurate momentum $\boldsymbol{Q}$ determined by the maximum in $\tilde{\chi}(\boldsymbol{q}, 0)$. (see, e.g., Grüner 1994). It is essential however that for fermionic dispersion as in equation (4), the enhancement of $\chi(\boldsymbol{q}, \omega)$ comes solely from near resonance in the denominator in equation (5), while the irreducible particle–hole susceptibility does not contain any information about strong magnetic fluctuations, and near the peak at $\boldsymbol{q} = \boldsymbol{Q}$ has a conventional Fermi-liquid form

$$\tilde{\chi}(\boldsymbol{Q}, \omega) = \frac{\alpha \tilde{\xi}^2}{1 - \mathrm{i}\omega/\tilde{\Gamma}_Q} \tag{6}$$

where $\tilde{\xi}$ is of the order of the lattice spacing, and $\tilde{\Gamma}_Q$ is an energy comparable to a fermionic bandwidth. We also assumed that there exist points on the Fermi surface which can be connected by $\boldsymbol{Q}$; otherwise, $\tilde{\Gamma}_Q$ would be infinite. It then follows from equation (5) that

$$\omega_{sf} \equiv \frac{c_{sw}^2}{2\gamma\xi^2} = \frac{\tilde{\Gamma}_Q \tilde{\xi}^2}{\xi^2}. \tag{7}$$

We see therefore that the key feature which makes the physics in the nearly antiferromagnetic Fermi liquid different from that in a conventional Fermi liquid is the presence of an energy scale $\omega_{sf} \propto \xi^{-2}$, which vanishes when the magnetic correlation length becomes infinite, and is obviously much smaller than the fermionic bandwidth as long as the susceptibility is strongly enhanced near $\boldsymbol{Q}$.

There are, however, several subtleties with the microscopic derivation of equation (3). First, in a RPA calculation, the fermions which contribute to the spin susceptibility are generally assumed to behave as free particles everywhere in momentum space, i.e., one completely neglects the incoherent part of the quasiparticle Green's function. For the calculations of the imaginary part of $\chi(\boldsymbol{q}, \omega)$, this assumption is justified as the integration over fermionic momentum is confined to a region where both fermions in the bubble have momenta near the Fermi surface. For these fermions, the incoherent part of the fermionic Green's function is irrelevant. The special points in momentum space for which both $\boldsymbol{k}$ and $\boldsymbol{k} + \boldsymbol{Q}$ are on the Fermi surface are usually referred to as hot spots (Hlubina and Rice 1995a, b). The real part of χ, on the other hand, comes from an integration over regions in momentum space which are far from the Fermi surface (see, e.g., Rickayzen 1980). In these regions, the incoherent part of $G(k, \omega)$ cannot be neglected as demonstrated in photoemission experiments (Wells *et al* 1995, Campuzano *et al* 1994). It is therefore likely that the actual position of the peak in $\mathrm{Re}\,\chi(q, \omega)$ is different from the RPA result and may well depend not only on doping but also on temperature.

Second, even though the computation of $\mathrm{Im}\,\chi(q, \omega)$ involves only coherent parts of the quasiparticle Green's functions, the RPA approach neglects possible strong vertex corrections to the polarization bubble. Explicit calculations show that these corrections are the strongest for the hot spots (Chubukov 1995, Altshuler *et al* 1995, Amin and Stamp 1996, Monthoux 1996). At small coupling they are obviously small to the extent that $g/\sqrt{\gamma\omega_{sf}} \leqslant 1$. At somewhat larger couplings, $g \geqslant \sqrt{\gamma\omega_{sf}}$, the relative vertex corrections which come from the integration near the Fermi surface scale as $[g^2\omega_{sf}\chi_Q/v_F^2]\log(C/\omega_{sf})$, where v_F is the Fermi velocity (we set the interatomic spacing $a = 1$). Now, if the damping term is computed self-consistently, then ω_{sf} by itself scales as g^{-2}, and the relative vertex correction depends on g only logarithmically. More explicitly, we have $g^{eff} = g[1 + 2\beta\log(C/\omega_{sf})]$ where β depends only on the shape of the Fermi surface near the hot spots, and is about $1/16$ for the experimentally measured Fermi surface at optimal doping (Altshuler *et al* 1995, Chubukov 1995). Notice that vertex corrections *increase*

the spin–fermion interaction ($g^{eff} > g$) and hence act in favour of magnetically induced, Eliashberg-type d-wave superconductivity. We see that β is small numerically, so if the logarithm is not large, the relative vertex correction can be neglected. In this case, the RPA analysis is approximately valid, and the imaginary part of the full susceptibility comes solely from the imaginary part of the particle–hole bubble. This is what we believe happens in the overdoped regime, which we associate with moderate values of $g \sim \sqrt{\gamma\omega_{sf}}$ in equation (1). However, we will argue below that the underdoped cuprates are described by a spin–fermion model with a somewhat larger ratio $g/\sqrt{\gamma\omega_{sf}}$, where new physics associated with Fermi-surface evolution appears. If we formally keep using the self-consistent approach, we find that at the onset of the Fermi-surface evolution, the logarithmic term overshadows the smallness of β, and the relative vertex correction is not small. In particular, for $g = g_{cr}^{(1)}$ defined below, $2\beta \log C/\omega_{sf} \approx \pi/4$. In this situation, the validity of the self-consistent RPA calculations of the damping term also becomes questionable.

From the above considerations we see that, although some properties of the NAFLM can be derived in Hubbard-based calculations, one has to be careful about using an RPA formalism in calculating the dynamical spin susceptibility. We therefore advocate a semi-phenomenological approach in which it is assumed that the spin susceptibility has the form of equation (3), with both ξ and γ taken from fits to the experimental data. We emphasize however that from a physical perspective, only ξ should be considered as an independent input parameter for equation (1). The damping of spin excitations due to the interaction with fermions is fully described by a spin–fermion model, and we will use the phenomenological form for ω_{sf} only because we are currently unable to self-consistently compute the spin damping at $g \gg c_{sw}\xi^{-1}$. We will, however, be able to estimate the value of the spin damping at very large couplings where the electronic structure develops the precursor of a spin-density-wave form.

We now consider the extent to which the physical properties of the normal state can be explained by the NAFLM.

3. Overdoped cuprates

We first discuss the overdoped cuprates. We define overdoped systems as those whose magnetic behaviour is such that in the normal state their uniform magnetic susceptibility, χ_0, is either independent of T or even increases with decreasing temperature, while the product $^{63}T_1T$ and the square of the ^{63}Cu spin-echo decay rate $(^{63}T_{2G})^2$ are linear in T above some temperature $T = T_{cr}$; see figure 2. At lower temperatures, T_1T becomes independent of T as in a conventional Fermi liquid. Examples of overdamped systems are $La_{2-x}Sr_xCuO_4$ for $x \geqslant 0.2$ and Bi 2212 and Tl 2201 for the appropriate oxygen content. The measured resistivity of overdoped cuprates is linear in T above some particular temperature $T = T_0 \sim T_{cr}$ and has a Fermi-liquid form $\rho \sim T^2$ at $T \ll T_0$; see figure 3. There is therefore just one crossover in system behaviour; see figure 4(a).

Our theoretical interpretation of the experiments on overdoped cuprates is based on the conjecture, first put forward by Barzykin and Pines (1995), that in these systems γ and c_{sw} are independent of temperature, while $\xi \leqslant 2$ for all $T > T_c$. This in turn implies that for overdoped cuprates

$$\omega_{sf}\xi^2 = \text{constant}. \tag{8}$$

Since the correlation length ξ (which, we remind the reader, is an input parameter for the NAFLM) does not exceed a few interatomic spacings, its temperature dependence is governed by fluctuations at lattice scales and is likely to be material dependent. NMR

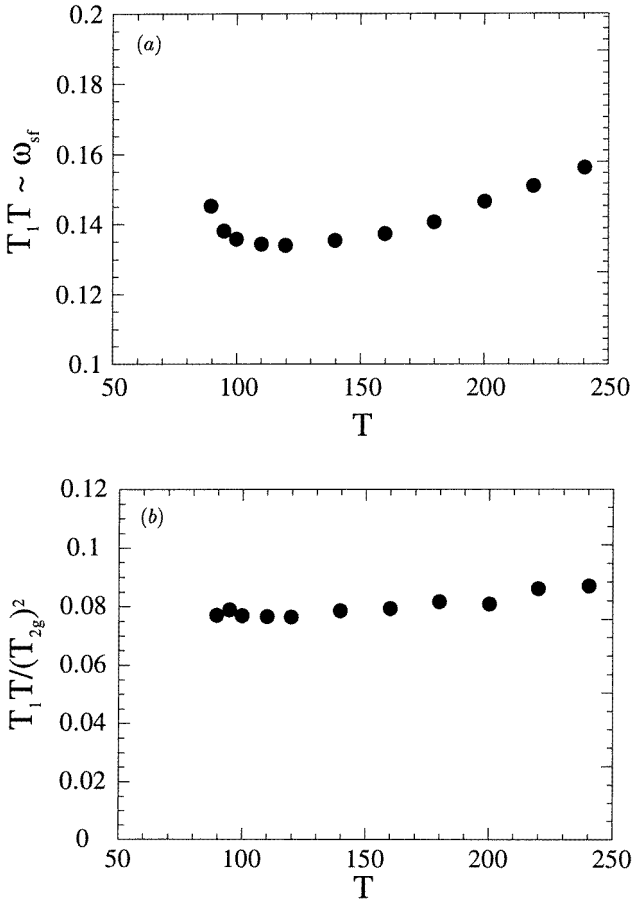

Figure 2. (a) The copper spin–lattice relaxation rate $^{63}T_1T$ as a function of temperature in slightly overdoped $Tl_2Ba_2CuO_{6+x}$ (Itoh *et al* 1994). Above 150 K $^{63}T_1T$ and, hence, ω_{sf} is proportional to T. At lower temperatures, $^{63}T_1T$ becomes T-independent as in a conventional Fermi liquid; (b) the ratio $^{63}T_1T/T_{2G}^2$, where T_{2G} is the spin-echo decay time. This ratio is practically independent of temperature, which is consistent with $z = 2$ scaling.

experiments tell us that for all of the compounds studied ξ^{-2} scales linearly with T, i.e., $\xi^{-2} = A + BT$, where A and B are (material-dependent) constants. More specifically, fits to the NMR data using equation (3) and the standard Shastry–Mila–Rice Hamiltonian for hyperfine interactions (Shastry 1989, Mila and Rice 1989), then yield (Thelen and Pines 1994):

$$^{63}T_1T \sim \frac{\omega_{SF}}{\alpha} \sim A + BT \tag{9}$$

and

$$(^{63}T_{2G})^2 \sim \left(\frac{1}{\alpha\xi}\right)^2 \sim A + BT \tag{10}$$

in agreement with the experimental results. The form of the susceptibility with γ and c_{sw} independent of temperature and $\xi^{-2} = A + BT$ is formally equivalent to that found in the $z = 2$ quantum-critical regime (Sachdev *et al* 1995). Clearly then, the T-dependence of all

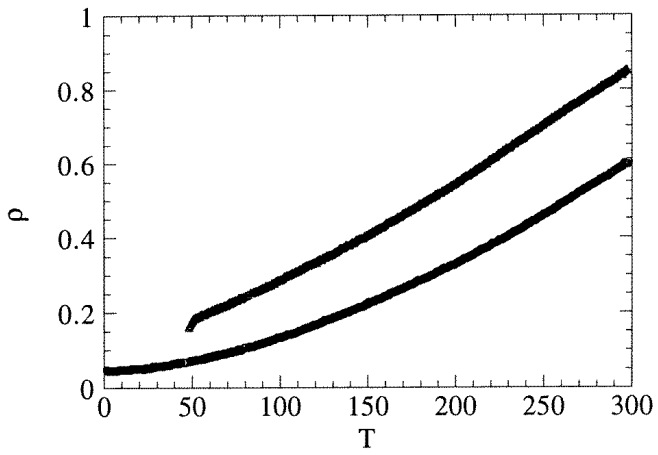

Figure 3. The resistivity as a function of temperature for two overdoped $Tl_2Ba_2CuO_{6+\delta}$ samples (Kitaoka *et al* 1991) with $T_c = 40$ K (upper curve) and $T_c = 0$ K (lower curve). The second sample is more heavily overdoped. Notice a pronounced T^2-behaviour in both samples at very low temperatures. The top curve clearly exhibits a crossover to $\rho \propto T$ somewhere between 100 and 200 K. The crossover temperature for the bottom curve is presumably larger than 300 K.

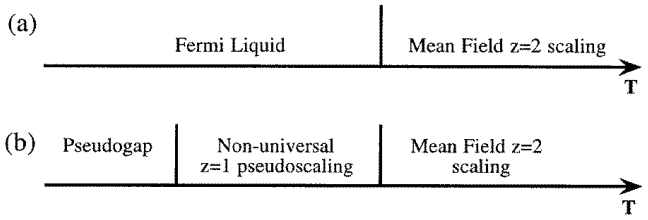

Figure 4. A sequence of crossovers in overdoped (a) and underdoped (b) cuprates.

observables associated with the spin susceptibility near (π, π) should be the same as for $z = 2$ scaling (Millis *et al* 1990). In particular, the measured ratio ${}^{63}T_1T/{}^{63}T_{2G}^2 \propto \omega_{sf}\xi^2$ is independent of temperature; see figure 2.

However, the analogy with $z = 2$ scaling is only a formal one. Although it works for observables, such as ${}^{63}T_1$ and T_{2G}, which probe the susceptibility near (π, π), the behaviour of the uniform susceptibility does not follow the scaling prediction, which is based on the assumption that spin fluctuations near $\boldsymbol{q} = \boldsymbol{0}$ and $\boldsymbol{q} = \boldsymbol{Q}$ are related. These are not related for the overdoped cuprates because their physics is dominated by the short-wavelength fluctuations. Indeed, above T_{cr}, $\chi_0(T)$ decreases with increasing T (NMR studies show that T_{cr} roughly corresponds to the temperature where $\chi_0(T)$ is a maximum). On the other hand, $1/N$ quantum-critical calculations yield $\chi_0(T)$ linearly increasing with T in the $z = 2$ regime (Sachdev *et al* 1995)†. As microscopic self-consistent (Eliashberg) calculations show (Monthoux and Pines 1993, 1994a), the decrease of susceptibility with increasing temperature is a natural consequence of lifetime effects, which act to reduce $\chi_0(T)$, and

† There is some uncertainty in the scaling prediction for $\chi_0(T)$ because $d = 2$ is an upper critical dimension for the $T = 0$ transition. Ioffe and Millis argued that the linear term in susceptibility could be of either sign (Ioffe and Millis 1995). The sigma-model-based calculations yield a positive linear-in-T term (Sachdev *et al* 1995).

which play an increasingly important role as the temperature increases.

The behaviour of the resistivity ρ is also non-universal, and, in principle, depends on the details of the electronic band structure. However, as we now show, in an argument which we shall see applies to the optimally doped cuprates as well as overdoped cuprates, $\rho(T)$ is in fact proportional to T over a wide temperature range due to the proximity of the quasiparticle Fermi surface to the magnetic Brillouin zone boundary. The reason for this is the following: according to the Drude formula, which is applicable to the overdoped cuprates because vertex corrections are small, the conductivity is proportional to the quasiparticle relaxation time averaged over the Fermi surface. The inverse relaxation time—the relaxation rate—is given by the imaginary part of the quasiparticle self-energy. As discussed above, the overdoped regime is associated with small to moderate values of the coupling constant. In this situation, second-order perturbation theory is likely to be valid. To second order in g we find by performing a summation over frequencies

$$\frac{1}{\tau_{\boldsymbol{k}}} = g^2 \int \frac{\mathrm{d}^2 k'}{2\pi^2} \,\mathrm{Im}\chi(\boldsymbol{k}-\boldsymbol{k}', \epsilon_k - \epsilon_{k'})[n(\epsilon_{k'} - \epsilon_k) + f(\epsilon_{k'})] \tag{11}$$

where $n(\epsilon)$ and $f(\epsilon)$ are Bose and Fermi distribution functions, respectively. The integration over $\boldsymbol{k}'$ can be split into an integration over $\epsilon_{k'}$ and an integration over surfaces of equal energy. For $\boldsymbol{k}$ located at the Fermi surface, the first integral can be evaluated analytically. The exact result is rather cumbersome, but to a good numerical accuracy it can be approximated as

$$\frac{1}{\tau_k} = \frac{\alpha c_{sw}^2 g^2}{8\gamma} \int_{FS} \frac{\mathrm{d}k'}{|\boldsymbol{v}_F(k')|} \frac{T^2}{\omega_{kk'}(\omega_{kk'} + \pi T)} \tag{12}$$

where $\omega_{kk'} = \omega_{sf}(1 + (\boldsymbol{k} - \boldsymbol{k}' - \boldsymbol{Q})^2\xi^2)$, and $\boldsymbol{k}$ and $\boldsymbol{k}'$ are two points at the Fermi surface. The same expression is indeed obtained in an ordinary Fermi liquid. In the latter case ω_{sf}, and hence $\omega_{kk'}$ are of the order of the Fermi energy which is large compared to T. One then immediately obtains $1/\tau_k \propto T^2$ for all k, i.e., the resistivity is proportional to T^2 as expected. In cuprates, however, ω_{sf} is much smaller than the Fermi energy; the dominant contribution to the integral over k' in (12) then comes from the regions where the distance between $\boldsymbol{k}'$ (at the Fermi surface) and $\boldsymbol{k} - \boldsymbol{Q}$ (at the 'shadow' Fermi surface) is minimal. Clearly, this minimal distance is zero at hot spots. Expanding near each of the hot spots, and performing the integration over k', we obtain

$$\tau_{\boldsymbol{k}} = \frac{4|v_F^{hs}|}{\alpha\pi g^2 \xi \sqrt{\omega_{sf}}} \frac{Q_k}{T^2} \tag{13}$$

where

$$Q_k = \sqrt{\omega_{sf}(1 + (\Delta k)^2\xi^2)}\sqrt{\pi T + \omega_{sf}(1 + (\Delta k)^2\xi^2)} \times \left[\sqrt{\omega_{sf}(1 + (\Delta k)^2\xi^2)} + \sqrt{\pi T + \omega_{sf}(1 + (\Delta k)^2\xi^2)}\right]. \tag{14}$$

Here Δk is the displacement of $\boldsymbol{k}$ from a nearby hot spot, and v_F^{hs} is the Fermi velocity at the hot spots. It is clear from equation (14) that τ_k is the largest in the regions of the Fermi surface where Δk is greater than the inverse correlation length, i.e., $(\Delta k)^2\xi^2 \gg 1$. These regions provide the dominant contribution to the conductivity, while the regions near hot spots contribute very little.

On averaging Q_k over the Fermi surface, one finds that $\langle Q_k \rangle$ is independent of T for $T < T_0$, and $\langle Q_k \rangle \sim T\sqrt{\omega_{sf}}\xi$ for $T > T_0$, where the crossover temperature, T_0, is given by

$$T_0 \approx \omega_{sf}(\Delta k_{max})^2\xi^2/2\pi. \tag{15}$$

The extra factor of $\frac{1}{2}$ in T_0 arises for numerical reasons. In general, one might expect that T_0 is of the order of the hopping integral, i.e., it is much larger than T over the whole experimentally probed range of temperatures. If this was the case, then the above reasoning would imply that the resistivity, $\rho \sim T^2\xi\sqrt{\omega_{sf}}/\langle Q_k\rangle$, would be roughly quadratic in T. However, photoemission experiments performed at or near optimal doping show that a substantial portion of the Fermi surface is located near the magnetic Brillouin zone boundary. It is not unreasonable to assume that this will be the case for the overdoped materials as well. In this case T_0 turns out to be much smaller than the hopping integral. For example, in $YBa_2Cu_3O_7$ on finds $T_0 \sim 15$ meV $\ll t \sim 250$ meV. In this situation $\langle Q_k\rangle \sim T\sqrt{\omega_{sf}}\xi$ starting already from relatively low temperatures and hence $\rho(T) \sim T$. Our numerical analysis performed for the parameters of the NAFLM chosen to fit NMR experiments in $YBa_2Cu_3O_7$ (Monthoux and Pines 1994a, Stojković 1996) shows that the resistivity is indeed roughly proportional to T over the temperature range $T \geqslant T_0 \sim 15$ meV. Hlubina and Rice (1995a, b) obtained $T_0 \approx 20$ meV with the same values of spin-fluctuation parameters, but somewhat different band parameters; hence they found $\rho \sim T^2$ up to higher temperatures. We emphasize once again that the linear behaviour of the in-plane resistivity is due to the fact that ω_{sf} is the smallest parameter in the problem, even though it scales as T: the typical $\omega_{sf}(\Delta k)^2\xi^2$ are smaller than $2\pi T$, but much *larger* than ω_{sf}. If $\omega_{sf} \propto T$ was comparable to πT above T_{cr}, then the resistivity would be proportional to $T^{1/2}$. Alternatively, if ω_{sf} was large and nearly temperature independent, as is the case in the more heavily overdoped cuprates, then ρ would be proportional to T^2, as seen experimentally. Notice also that, since $\langle Q_k\rangle \propto T\sqrt{\omega_{sf}}\xi$ above T_0, the resistivity does not depend on the spin damping and hence on $\omega_{sf}\xi^2$. We make use of this fact below when we discuss the behaviour of the resistivity at smaller doping concentrations where γ acquires a strong T-dependence.

It is also instructive to discuss the temperature dependence of the average scattering rate, $\langle\langle 1/\tau_k\rangle\rangle$ for $T > T_0$. Under the same conditions as above, $Q_k \propto T(\Delta k)$, and the integration over Δk yields $\langle 1/\tau_k\rangle \propto T\log((\Delta k_{max})^2/\omega_{sf})$. However, the T-dependence coming from the logarithm is rather weak and to a good accuracy $\langle 1/\tau_k\rangle \propto T$. Therefore, we see that at $T > T_{cr}$, inverting the average of τ_k and averaging $1/\tau_k$ yields a similar linear-in-T dependence of the resistivity.

4. Underdoped cuprates

We now turn to the key issue of the paper, the behaviour of the underdoped materials. These may easily be distinguished from overdoped materials on the basis of their low-frequency magnetic behaviour, depicted in figure 1 (Barzykin and Pines (1995); see also Sokol and Pines (1993)). Examples of underdoped cuprates include $YBa_2Cu_4O_8$, $YBa_2Cu_3O_{6+x}$, for $x \leqslant 0.93$, $La_{2-x}Sr_xCuO_4$ for $x \leqslant 0.2$, $Bi_2Sr_2CaCu_2O_{8-\delta}$ (Bi 2212) compounds with adjusted oxygen partial pressure during annealing and Hg 2223. As seen in figure 1, underdoped cuprates exhibit two different crossovers in the normal state at the temperatures T_{cr} and $T_* < T_{cr}$. Above T_{cr} their behaviour is similar to what has been observed for overdoped systems: it is non-universal, but the resistivity is approximately linear in T, and the ratio ${}^{63}T_1T/{}^{63}T_{2G}^2$ is independent of T, as shown for $YBa_2Cu_4O_8$ in figure 5(a). The linear behaviour of the resistivity persists below T_{cr} with only a subtle change in slope; however, the magnetic properties change rather drastically below T_{cr}. As shown in figure 5(b) for $YBa_2Cu_4O_8$, ${}^{63}T_1$ becomes almost independent of T, while T_{2G} becomes proportional to T in such a way that the ratio ${}^{63}T_1T/{}^{63}T_{2G}$ is independent of T. Using equations (9) and (10), we find that in this T-range the inverse correlation length and ω_{sf}

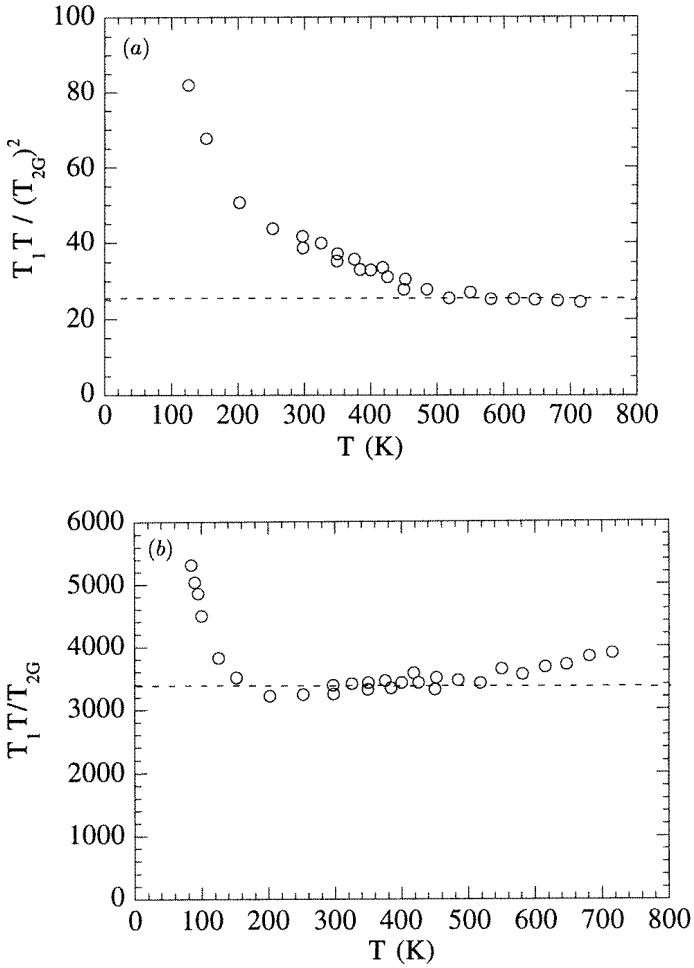

Figure 5. (a) The ratio $^{63}T_1T/T_{2G}^2$ as a function of temperature in underdoped $YBa_2Cu_4O_8$. As in overdoped samples, this quantity becomes independent of temperature above $T_{cr} \sim 450$–500 K; (b) $^{63}T_1T/T_{2G}$ as a function of temperature for the same compound. This ratio is almost a constant in the temperature interval $T_* < T < T_{cr}$, which is a signature of the $z = 1$ scaling regime. In both graphs, the dashed line is a guide to the eye.

both increase linearly with increasing T, in such a way that $\omega_{sf}\xi = \text{constant}$, implying $z = 1$ behaviour. The uniform susceptibility is also proportional to T below T_{cr}. Finally, at even lower temperatures, $T < T_*$, the system enters into a regime where the correlation length becomes independent of T, $^{63}T_1T \propto \omega_{sf}$ *increases* as T decreases further towards T_c ($^{63}T_1T$ displays a minimum at $T \sim T_*$), and both the uniform susceptibility and $1/^{63}T_1T$ fall off sharply with decreasing T. This regime has been called a 'pseudogap regime' as the behaviour of, e.g., $\chi_0(T)$, is, at first sight, reminiscent of the behaviour of systems which display a true quasiparticle energy gap, such as a superconductor or an ordered antiferromagnet. However, the curvature of the fall-off in $\chi_0(T)$ is opposite to that found below T_c in conventional superconductors (Slichter 1994).

We see therefore that the new features of the underdoped cuprates, as illustrated in

figure 1, are (i) the crossover from $z = 2$ to $z = 1$ behaviour at T_{cr}, and (ii) the crossover to pseudogap behaviour at an even lower value of T_*. This sequence of crossovers is shown schematically in figure 4(b).

Before presenting our scenario for these crossovers, we briefly review the results of the σ-model-based studies of the low-temperature crossovers between different *universal* scaling regimes in disordered antiferromagnets (Chakravarty *et al* 1988, 1989, Chubukov *et al* 1994b, Sachdev *et al* 1995). In the disordered state at $T = 0$, the antiferromagnets possess a gap, Δ, in the excitation spectrum. If the damping term at $T = 0$ is much smaller than Δ, then, as T increases, the system experiences a crossover from a quantum-disordered regime with an exponential ($e^{-\Delta/T}$) behaviour of observables, to the $z = 1$ quantum-critical regime. If, in contrast, at $T = 0$, $\gamma \gg \Delta$, then the sequence of crossovers with increasing T is from a quantum-disordered regime with a *power-law* behaviour of observables to the $z = 2$ scaling regime with overdamped spin fluctuations, and finally, at even larger T, to the $z = 1$ scaling regime in which typical magnon frequencies and the damping term both scale linearly with T.

We see that for all ratios of γ/Δ the prediction of the sigma-model-based studies is that $z = 1$ behaviour always occurs at higher temperatures than $z = 2$ behaviour. This is simply due to the fact that $z = 1$ scaling requires that typical frequencies should be larger than the $T = 0$ value of the damping term. Experimentally, however, the situation is the opposite: in underdoped cuprates, one observes purely relaxational behaviour at high enough temperatures, and $z = 1$ scaling at lower temperatures (Barzykin 1996). The crossover at low T from the quantum-disordered behaviour to the $z = 1$ quantum-critical behaviour is more consistent with the observations around T_*, but again, the opposite curvature of the fall-off in $\chi_0(T)$ compared to the sigma-model prediction indicates that the crossover at T_* involves not only localized spins, but also electronic degrees of freedom.

Monthoux and Pines (1994b) proposed that the reversed behaviour observed in underdoped cuprates below T_{cr} is due to the strong temperature variation of the damping term γ in the dynamical spin susceptibility, which in turn gives rise to the anomalous temperature dependence of ω_{sf}. This temperature variation is neglected in the σ-model approaches which assume that throughout the whole temperature range of interest, the input parameters in the dynamical susceptibility retain the same values as at $T = 0$. Below we relate the temperature variation of the damping rate to the formation of a precursor to a spin-density-wave state which in turn causes the evolution of the quasiparticle Fermi surface with temperature and doping concentration. We argue that the two crossover scales T_{cr} and T_*, observed in underdoped cuprates indicate the onset and the end point of this Fermi-surface evolution, respectively. At T_{cr}, the onset of the crossover, the quasiparticle residue along the Fermi surface develops a minimum at hot spots; at T_*, the end point of the crossover, the system actually begins to lose pieces of the Fermi surface. In other words, we argue that there exists only one extended crossover in system behaviour in which the electronic structure gradually develops the features of a precursor to a spin-density-wave state. In this scenario, $z = 1$ scaling in the temperature range $T_* < T < T_{cr}$ is just an intermediate asymptotic in the extended crossover region, rather than an extension of the $z = 1$ quantum-critical scaling observed in the intermediate T-range right at half-filling.

Fermi-surface evolution with doping at fixed T has been observed in photoemission experiments on YBCO (Liu *et al* 1992) and more recently in experiments on Bi 2212 (Marshall *et al* 1996, LaRosa *et al* 1996). These experiments demonstrated that while near optimal doping the hole Fermi surface is large and encloses an area consistent (to within the accuracy of the measurement) with the Luttinger theorem, the measured Fermi surface in underdoped materials loses pieces near $(\pi, 0)$ and symmetry-related points. Recent

experiments by Ding *et al* (1996) have shown that the same effect occurs at a fixed doping as the temperature is lowered. The transformation of spectral weight from the low-frequency part of the spectrum to higher energies upon approaching half-filling has been observed by Timusk and his collaborators in optical experiments on the planar conductivity $\sigma(\omega, T)$ (Puchkov *et al* 1996); the same experiments also demonstrated that there is no spectral weight transformation at optimal (and larger) doping.

Recently, one of us has considered Fermi-surface evolution with increasing spin–fermion coupling constant in the NAFLM at $T = 0$ (Chubukov *et al* 1996). It was found that as g increases, the quasiparticle residue, Z, near hot spots decreases. The decrease of Z becomes appreciable when the coupling constant g exceeds a critical value $g_{cr}^{(1)} \sim v_F/(\omega_{sf}\chi_Q \log(C/\omega_{sf}))^{1/2}$. Notice that $g_{cr}^{(1)}$ vanishes logarithmically when the correlation length becomes infinite. As g increases even further, the quasiparticle Fermi surface undergoes a substantial evolution in the process of which parts of the Fermi surface near the corners of the magnetic Brillouin zone (where the hot spots are located when $g \to 0$) move away and, simultaneously, there appear two distinct peaks in the density of states—the precursors of the valence and conduction bands. This evolution of the Fermi surface occurs in the range of g-values of the order of the upper cut-off in the staggered spin susceptibility, $g \sim g_{cr}^{(2)} \sim C > g_{cr}^{(1)}$. It was further argued that as the system approaches half-filling, the effective coupling constant g increases, while both $g_{cr}^{(1)}$ and $g_{cr}^{(2)}$ decrease, so in varying the ratio g/g_{cr} one in fact varies the doping concentration.

Suppose now we fix the doping and vary the temperature. As the temperature decreases, the inverse correlation length and the damping rate decrease, and, hence, both critical values of g go down ($g_{cr}^{(1)}$ goes down chiefly because of the decrease of damping, while $g_{cr}^{(2)}$ goes down chiefly because of the decrease of ξ^{-1}). As a result, even if the effective coupling constant weakly depends on T, the ratios, $g/g_{cr}^{(1)}$ and $g/g_{cr}^{(2)}$ still increase with decreasing T. This in turn implies that, as the temperature is lowered, one should observe crossovers in the system behaviour analogous to those described above. These crossovers will occur at temperatures at which $g_{cr}^{(1,2)}(T)$ become equal to a given g. It seems natural to associate the temperature at which $g_{cr}^{(1)}(T_{cr}) = g$ with T_{cr}, and the temperature at which $g_{cr}^{(2)}(T_*) = g$ with T_*.

We now consider the conditions under which the system exhibits $z = 1$ scaling behaviour below T_{cr}. Scaling with $z = 1$ requires that typical frequencies and momenta be of the order of T. In the NAFLM, this will be the case if both ξ^{-1} and ω_{sf} scale linearly with T, which in turn implies that $\tilde{\Gamma}_{Q_1}^{-1}$ and γ should exhibit the same linear dependence on T as does ξ^{-1} (Chubukov *et al* 1994b, Monthoux and Pines 1994b). Thus one should have between T_{cr} and T_*

$$\gamma \sim \tilde{\Gamma}_{Q}^{-1} \sim \xi^{-1} = a + bT \tag{16}$$

while, as we have seen, above T_{cr}, γ and $\tilde{\Gamma}_{Q}^{-1}$ are independent of T. As we discussed above, the temperature-dependent inverse correlation length should be considered as an input parameter in the NAFLM because if one attempts to compute the real part of the spin susceptibility (from which ξ is inferred) in RPA-type calculations, one would need to know the exact form of the fermionic Green's function far away from the Fermi surface. Hence, the linear-in-T dependence of ξ^{-1} below T_{cr} follows from the fit to the NMR data of $^{63}T_1$ and T_2 (Barzykin and Pines 1995). In contrast, the linear-in-T dependence of γ has to be obtained *within* the NAFLM approach, although at the moment we do not have a clear recipe for calculating it.

It is essential however not to confuse the intermediate $z = 1$ scaling regime in underdoped cuprates with the $z = 1$ quantum-critical scaling in pure antiferromagnets.

In the latter case, the damping of spin excitations is due to the interaction between spin fluctuations, and one can argue quite generally that it should be linear in T when $\xi^{-1} \propto T$ (Chubukov *et al* 1994b). However, the estimates of Barzykin and Pines (1995) show that at doping concentrations where experiments on underdoped cuprates have been performed, the purely magnetic damping is too small to account for the experimental data. In other words, in the $z = 1$ regime in underdoped cuprates, the damping primarily comes from the interactions with fermions (this is why we describe this regime as 'pseudoscaling'). At the same time we expect that below T_{cr}, lattice corrections are not large. Then, once the $z = 1$ form of the dynamical susceptibility for $\boldsymbol{q}$ near $\boldsymbol{Q}$ is obtained, one can use the universal scaling forms obtained for $z = 1$ quantum-critical scaling for both uniform and staggered susceptibility. In particular, this implies that the static uniform susceptibility should be linear in T, in agreement with experiment. This, we recall, is different from the $z = 2$ regime at higher temperatures, where the physics is dominated by short-wavelength fluctuations and one could only formally use the predictions of the $z = 2$ quantum-critical scaling to obtain the T-dependence of observables associated with the dynamical susceptibility near $\boldsymbol{Q}$; however, the temperature dependence of the uniform susceptibility was very different from the scaling prediction.

We note in passing that the fact that spin damping is chiefly due to the interaction with fermions may resolve an apparent discrepancy between the values of the spin-wave velocity in $YBa_2Cu_3O_{6.63}$ extracted from a linear fit to the uniform susceptibility and from the ratio ${}^{63}T_1T/T_{2G}$ (Millis and Monien 1994, Chubukov *et al* 1994a). Indeed, the slope of the uniform susceptibility $\chi_0 \propto T$ is finite in the absence of spin damping. It does not change much with doping compared to its value at half-filling, and therefore is likely to only weakly depend on γ. Accordingly, the fit for $\chi_0(T)$ yields a c_{sw} which is slightly larger than that found at half-filling, which agrees with analytical calculations at small doping (Chubukov and Frenkel 1992, Schulz and Zhou 1995). At the same time, the NMR relaxation rate is proportional to the damping, and for this quantity it is essential to know where the damping comes from. A fit to NMR data using the σ-model results yields a spin-wave velocity which is a few times smaller than at half-filling. If instead, we extract c_{sw} from the NMR data using Fermi-liquid damping, we obtain a spin-wave velocity larger by a factor γ_{FL}/γ_{sf} which better agrees with the velocity extracted from the susceptibility data.

We consider next the behaviour of the resistivity. Experimentally, it is linear in T both above and below T_{cr} with about the same slope. On the theoretical side, we have seen above that the resistivity is linear in T as long as over substantial portions of the Fermi surface, T is larger than T_0. This result holds approximately even when γ acquires a substantial T-dependence, provided that the vertex corrections are not too large. More importantly, the decrease of the spectral weight near the hot spots has little effect on the conductivity, simply because the dominant contributions to conductivity come from the pieces of the Fermi surface which are relatively far from hot spots. A careful inspection of the measured shape of the Fermi surface in YBCO compounds (Gofron *et al* 1994, Campuzano *et al* 1994) and the fits to NMR measurements (Barzykin and Pines 1995), shows that in the underdoped cuprates T_0 is also substantially smaller than T_{cr}. In the intermediate, $z = 1$ scaling regime, γ acquires a temperature dependence, and T_0 becomes a function of T, scaling as $T_0 \sim T^{-1}$. However, the slope is rather small and everywhere in the $z = 1$ regime T_0 remains smaller than T, i.e., the resistivity remains linear in T, in good agreement with the experimental results in, e.g., $YBa_2Cu_4O_8$ (Bucher *et al* 1993; see figure 6). The two temperature scales (T and $T_0(T)$) become comparable to each other at a temperature which in $YBa_2Cu_4O_8$ is rather close to T_* (T_* is roughly 220 K and $T_0(T_*) \sim 250$ K). We emphasize, however, that the behaviour of resistivity in our model is non-trivial and depends on the shape of the

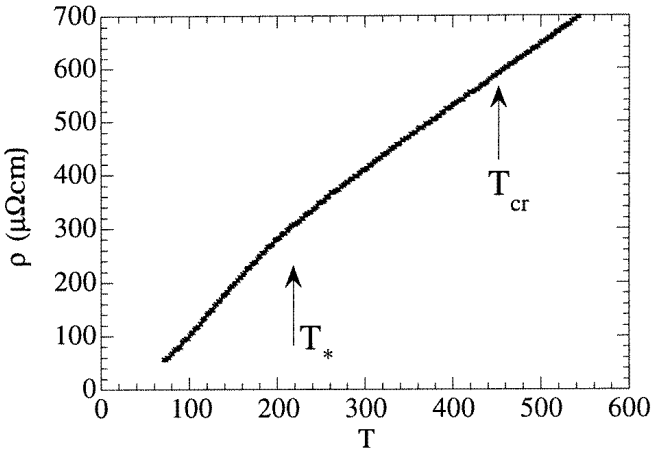

Figure 6. The resistivity along the a-axis for underdoped $YBa_2Cu_4O_8$. Notice that the linear behaviour starts at around T_*, and the minor change of slope at $T = T_{cr}$. Below T_*, the resistivity falls off rapidly with decreasing T.

quasiparticle band structure. Hence our argument, based on $YBa_2Cu_4O_8$, that the crossover in resistivity occurs near T_* need not necessarily apply to all high-T_c compounds.

We now turn to the crossover at $T = T_*$. Barzykin and Pines (1995) associated this crossover with the transformation from the $z = 1$ quantum-critical to the $z = 1$ quantum-disordered regime. We have associated the same crossover with the development of the precursors of the spin-density-wave state in the electronic structure. We now show that the two identifications of the crossover at T_* are in fact complementary. We note first that in order to obtain predominantly $z = 1$ behaviour for the observables in the quantum-disordered regime, one must get rid of the Landau damping term in the dynamical staggered susceptibility. Otherwise, the system at the lowest T will necessarily cross over to the quantum-disordered $z = 2$ regime, where, e.g., $^{63}T_1T \propto \omega_{sf} =$ constant, which does not agree with the data below T_* (we recall that the measured $^{63}T_1T$ *increases* as the temperature is lowered below T_*). What happens when the quasiparticle spectrum develops a pseudogap near the corners of the Brillouin zone? At first glance, one might argue that the spin damping must become exponentially small in T at low temperatures because the former hot spots which chiefly contributed to the damping in the overdoped regime disappear. However, this argument is incorrect because in a process of the Fermi-surface evolution towards small pockets, there appear new hot spots which survive even in a situation in which the spin-density-wave structure of the electronic states is already developed.

The actual reason that the damping term goes down sharply when pockets are formed relates to the form of the fully renormalized spin–fermion vertex: in the spin-density-wave state with long-range order this vertex vanishes identically for the bosonic momentum Q because of the Ward identity (Adler principle); thus, the damping term in the spin susceptibility in fact scales as $\mathrm{Im}\,\chi^{-1}(q,\omega) \propto \mathrm{i}\omega(q-Q)^2$ (Sachdev 1994, Schrieffer 1995, Sachdev *et al* 1995). In the precursor to the spin-density-wave state, there is no precise requirement that the full vertex at Q should vanish; however, explicit calculations show that vertex corrections (which in the $g \gg C$ limit have a negative sign) almost completely cancel the bare interaction g such that the fully renormalized vertex turns out to be small compared to g by a factor of $(g_{cr}^{(2)}/g)^2$ (Chubukov *et al* 1996). Very similar arguments have

been previously provided by Schrieffer (1995).

A nice feature of the large-g limit in the spin–fermion model is that the smallness of $g_{cr}^{(2)}/g$ not only yields a small vertex but also allows one to compute the full vertex and full fermionic Green's function by expanding in powers of $g_{cr}^{(2)}/g$. This in turn allows one to compute fermionic damping: one just substitutes the full Green's functions and full vertex into the particle–hole bubble. We have performed this calculation and found that the damping term contains a small factor $(g_{cr}^{(2)}/g)^4$. Since the ratio $g/g_{cr}^{(2)}$ decreases with T, the damping term rapidly decreases with decreasing temperature, and at low temperatures becomes smaller than $c_{sw}\xi^{-1}$ which is the energy scale of a gap in the spin susceptibility. In this situation, the system should display predominantly $z = 1$ quantum-disordered behaviour, in agreement with the data below T_*. In particular, $\omega_{sf} \propto 1/\gamma$ should *increase* as T decreases which in turn leads to the *increase* of ${}^{63}T_1T$ with decreasing temperature. In addition, below T_*, the resistivity scales as $\rho \propto T^2/\omega_{sf}$, i.e., it rapidly (faster than T^2) decreases with decreasing T. This rapid decrease of ρ is also consistent with the experimentally observed behaviour of the resistivity (see figure 6). Finally, at even lower temperatures, the system should, in principle, undergo a crossover to $z = 2$ quantum-disordered regime, but the crossover temperature is probably lower than T_c which implies that this crossover cannot be observed experimentally.

The reduction of the spin–fermion interaction vertex also affects the low-T behaviour of the uniform susceptibility. In a fully developed spin-density-wave state with long-range order only the longitudinal susceptibility has a (doping-dependent) Pauli contribution, associated with a finite density of holes; the transverse susceptibility $\chi_\perp$ at finite doping remains virtually the same as at half-filling because the Pauli contribution to $\chi_\perp$ is reduced by vertex renormalization (Chubukov and Frenkel 1992, Chubukov and Musaelian 1995). Specifically, we have

$$\chi_\perp^{(T=0)} = \frac{Z_\chi}{4J} + \mathrm{O}(x) \qquad \chi_{zz}^{(T=0)} = \frac{1}{2}\chi_\rho \tag{17}$$

where x is the doping concentration, Z_χ is the quantum renormalization factor, and for a Fermi surface consisting of small pockets near $(\pi/2, \pi/2)$, one finds $\chi_\rho = \sqrt{m_1 m_2}/2\pi$, where m_1 and m_2 are the two effective masses for fermionic dispersion. In order to estimate χ_ρ, we use the photoemission data for the oxychloride $Sr_2CuO_2Cl_2$ (Wells *et al* 1995, LaRosa *et al* 1996). These data yield $m_1 \approx m_2 \approx 1/2J$, or $\chi_\rho \approx 1/4\pi J$. In the preformed spin-density-wave state the transverse and longitudinal susceptibilities are indistinguishable, so

$$\chi_u(T \to 0) \equiv \frac{1}{3}\chi_\perp + \frac{1}{3}\chi_{zz} = \frac{Z_\chi}{12J} + \frac{1}{6}\chi_\rho. \tag{18}$$

Consider further the underdoped cuprates with a magnetically disordered ground state and a small Fermi surface. As the correlation length saturates to a finite value as $T \to 0$, the magnetic part of the susceptibility vanishes ($Z_\chi \to 0$). We are thus left with only the Pauli contribution, i.e., $\chi_u = (1/3)(\chi_\rho/2) \approx 1/24\pi J$. We emphasize that the factor 1/3 in χ_u is due to the spin–fermion vertex reduction, i.e., the same effect as yields a reduction in the spin damping. Reinserting the $(g\mu_B)^2$ factor in χ_u we obtain $\chi_u/\mu_B^2 \sim 0.4$ states eV^{-1} per Cu atom which is some six times smaller than $\chi_u/\mu_B^2 \geqslant 2.6$ states eV^{-1}, the result for an optimally doped sample such as $YBa_2Cu_3O_7$, or for overdoped samples. We see therefore that the uniform susceptibility in the preformed spin-density-wave state with a small Fermi surface is much smaller than that found at larger doping levels with large Fermi surfaces, despite the fact that the Pauli susceptibility in 2D does not depend on p_F and is the same for large and small elliptical Fermi surfaces.

5. Optimal doping

Optimally doped cuprates are frequently defined as those which, within a given family, exhibit the highest T_c. Examples of optimally doped materials include, e.g., $YBa_2Cu_3O_{6.93}$ and $La_{1.85}Sr_{0.15}CuO_4$. Photoemission experiments on YBCO systems and the bismuthates have shown that these materials have a large Fermi surface in close analogy with overdoped cuprates. At the same time, from a magnetic perspective, optimally doped materials are members of the underdoped family, in that their uniform susceptibility exhibits the same crossovers at T_{cr} and T_* as are seen in other underdoped materials. The analogy between underdoped and optimally doped materials has also been observed in photoemission experiments on Bi 2212 (Ding *et al* 1996): in both types of material, there exists a pseudogap in the normal state which disappears at $T \sim T_{cr}$ which is substantially larger than T_c.

Another feature of optimally doped materials is that their resistivity continues to be linear in T down to $T \sim T_c$. According to the arguments that we have given above, such linearity above T_0 reflects both the closeness of the Fermi surface to the magnetic Brillouin zone boundary and the particular spin-fluctuation spectrum. It is tempting to conjecture that in the optimally doped cuprates the Fermi surface is rather flat in such a way that $(\Delta k_{max})^2$ is minimized, which in turn yields a minimum in T_0 as a function of doping. This flattening of the Fermi surface would also give rise to stronger vertex corrections which, as we recall, act to increase the spin–fermion interaction vertex and hence T_c, before any changes in the Fermi-surface topology take place.

6. Summary

We have proposed a specific scenario for the temperature crossovers in the overdoped and underdoped cuprates. We considered a nearly antiferromagnetic Fermi-liquid model and argued that in the overdoped regime the spin damping, $\gamma \propto (\omega_{sf}\xi^2)^{-1}$, is independent of temperature over the entire experimentally probed T-range. In this situation the magnetic behaviour is described by a mean-field $z = 2$ dynamical susceptibility. We argued that the resistivity has a crossover from a Fermi-liquid-like T^2-behaviour at $T < T_0$ to a linear-in-T behaviour for $T > T_0$; the crossover temperature T_0 is low due to the proximity of the Fermi surface to the magnetic Brillouin zone boundary. We further argued that in order to account for the experimentally measured sequence of crossovers in underdoped cuprates from the Fermi-liquid, $z = 2$ behaviour at high temperatures to the $z = 1$ scaling behaviour at intermediate T, and to the pseudogap behaviour at even lower T, one has to take into consideration the thermal variation of the damping rate of spin excitations, γ. We argued that the primary source of the variation of γ with T in the underdoped cuprates is the thermal evolution of the quasiparticle dispersion near the Fermi surface produced by the precursor of a spin-density-wave state. Within our approach, we found consistency with the experimental data on resistivity, NMR relaxation rates and uniform susceptibility in underdoped cuprates (for a discussion of the application of the NAFLM to the Hall effect measurements see Stojković 1996). We emphasize, however, that here we only discuss a possible scenario of how the damping evolves with T. At present, we can compute this damping either for very small or very large values of the coupling constant. Direct calculations of $\gamma(T)$ within the spin–fermion model at intermediate couplings are clearly called for.

A final note. Our scenario is in contradiction with the proposals (Emery and Kivelson 1995, Randeria *et al* 1994, Ding *et al* 1996) that the physical origin of the pseudogap behaviour is the precursor to the d-wave pairing state. Both this and our scenario imply that the quasiparticle gap near $(0, \pi)$ observed in photoemission measurements in the normal

state of underdoped cuprates should not change as the system becomes superconducting, in agreement with the data. In the precursor to the d-wave pairing scenario, the superconducting gap is already preformed by superconducting fluctuations, while in our scenario there exists a preformed spin-density-wave gap near $(0, \pi)$.

The photoemission data of Ding *et al* (1996) have been interpreted as evidence in favour of a preformed d-wave gap. They measured a gap as a shift of the midpoint of the leading edge of the photoemission spectrum, and found that the shift, which is of the order of 20 meV, has the same k-dependence as the d-wave gap in the superconducting state. In contrast, Marshall *et al* (1996) estimated the gap from the position of the maximum in the spectral function. They found a much larger gap of about 0.2 eV for the 60 K superconductor, which is fully consistent with spin-density-wave scenario (in a fully developed spin-density-wave state, the gap near $(0, \pi)$ is about $2J \sim 0.25$ eV). In the absence of a theory for the lineshape, it is difficult to say with confidence which interpretation of the data is correct. We note, however, that no preformed gap has been observed in overdoped cuprates. While the magnetic scenario leads naturally to the prediction that there should be no sign of a pseudogap in overdoped cuprates (superconductivity occurs before a spin-density-wave precursor can develop), it is not clear why precursors to the d-wave pairing state should disappear in overdoped materials. Finally, we note that the magnetic scenario correctly describes the whole sequence of crossovers in the normal state including the crossover at T_{cr} which can easily be much larger than T_c (e.g., $T_{cr} \approx 6T_c$ in $YBa_2Cu_4O_8$). Moreover, NMR experiments on $YBa_2Cu_3O_7$, $YBa_2Cu_4O_8$ and $La_{1.85}Sr_{0.15}CuO_4$ suggest that the magnetic correlation length is approximately the same at T_{cr}, $\xi(T_{cr}) \sim 2a$, in all of these compounds, implying yet another connection between the magnetic and the pseudogap behaviour. It also seems unlikely that one could explain the crossover to $z = 1$ scaling as due to a precursor of the d-wave pairing. We therefore believe that the pseudogap has a magnetic rather than a superconducting origin.

Several researchers (see, e.g., Millis and Monien 1994) have argued that the pseudogap behaviour in the underdoped cuprates is due to an exchange coupling, $J_\perp$, between the bilayers found in a unit cell. Indeed, strong bilayer coupling leads to singlet configurations of adjacent spins which gives rise to a gap in the spin excitation spectrum. In an insulator, however, one needs a rather large $J_\perp$ ($\sim 2.5J$) to produce the single configuration between adjacent spins in the bilayers, while recent experiments find $J_\perp \sim 0.1J$ (Keimer *et al* 1996). For doped materials, Ioffe *et al* (1994) argued that the effective bilayer coupling scales as $J_\perp^{eff} \propto J_\perp \chi^2(Q)$, i.e., it is enhanced if susceptibility is strongly peaked at Q. This effect is certainly present at optimal doping; however, we argued above that in the underdoped cuprates (where a pseudogap has been observed), the enhancement of susceptibility is compensated by vertex corrections. In this situation, J^{eff} should be of the same order as the bare coupling and is unlikely to give rise to a spin gap unless one assumes that there is a spin–charge separation (Millis *et al* 1996, Ubbens and Lee 1994). Our point of view is that the pseudogap behaviour is an intrinsic property of a single CuO_2 layer as the data on the spin–lattice relaxation rate, the uniform susceptibility, and $\xi(T)$ in the single-layer $La_{2-x}Sr_xCuO_4$ materials display the same sequence of crossovers as seen for the bilayer $YBa_2Cu_3O_{6+x}$ materials.

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